

The influence of partial oscillations on the vibratory displacement of grainy material under different frequency vibration mode

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Received: 15 March 2020; accepted: 30 May 2020

ABSTRACT

Vibrational transportation-technologic (VTT) process is a dynamically sensitive process, in which many physically different components are involved: vibrodrive, elastic system, working member (absolutely or finitely rigid), various grainy loads. The interactions of those components define the behavior of grainy material on the surface of working member. As a main defining factor of the accuracy of the process, precise transmission of vibration to the working member is of particular importance. In the manuscript, the special dynamical model of the VTT system and corresponding mathematical model of the VTT process are presented, where the movement of grainy material is described under the conditions of spatial vibration of the working member, which is caused by possible errors in the manufacture and assembly of the vibration machine. Mathematical modeling has been carried out under continuous operating conditions (amplitude, frequency) of electromagnetic resonance vibrofeeder, when simultaneously alternating amplification of separate non-working spatial vibration and revelation of its impact on the process takes place. The results of modeling in the form of graphs and the influence of non-working vibrations on the law of the material displacement are presented in the manuscript.

Keywords: Vibratory displacement, Partial oscillations, Grainy material, Resonant vibration, Mathematical modeling, Vibratory Feeder.

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1. Introduction

Vibrational transportation-technologic (VTT) machines are widely used in many fields of manufacturing in the world for vibrational processing and transportation of different kinds of materials [1-11].

VTT machines (drawing.1c) are used in construction, mining, agriculture, chemical and confectionery manufactures and others, therefore, the dimensions of machines vary, as well as their power, shapes of working member and types of vibro-drive and elastic systems.

VTT process is a dynamically sensitive one, in which many physically different components are involved: vibro-drive, elastic system, working member (absolutely or finitely rigid), various grainy loads [10, 12-17]. The interactions of those components define the behaviour of grainy

material on the surface of working member.

Due to various construction, installation or manufacturing errors in spring vibrating machines [12, 17], Also, due to the specificity of the springs, there is a deviation of the excitation force from the calculating direction. As a result, spatial oscillations occur along with the operating oscillations of the working member [17-19], which, in the case of amplification (e.g., under resonance conditions), can have a significant impact on the vibrational displacement pattern of loose material.

Based on the above, it is advisable to develop a generalized (spatial) mathematical model of a loaded vibrational technological machine, where the possible cinematic and dynamic connections and interactions between the constituent components will be reflected. Such an approach will allow us to investigate through modeling

the influence of a wide range of parameters on the regularity of movement of material (technological load).

2. Spatial dynamical model of vibrational displacement of grainy material

A spatial dynamical model of vibrational transportation of grainy material has been developed to investigate this problem (Fig. 1). A three-mass (vibro exciter – working member – grainy load) dynamical model of the vibro-feeder is shown in the figure, where the following assumptions are adopted:

- Working member, as an asymmetrical rigid body, performs spatial (rotational-linear) vibrational motion;
- Grainy material is considered as a solid body, which will be equipped with conditional elastic-damping elements, which characterize (describe) the rheological properties of grainy material;
- The movement of grainy load is considered in three linear directions;
- The third mass - vibroexciter is considered immobile, from which the vibration is transmitted to the working member in a single direction;
- The excitation force is not transmitted precisely to the center of gravity of the working member, but is missed due to various permissible structural and physical inaccuracies [5, 19], which generates force projections and moments towards the center of gravity.

The working member ($O_1x_1y_1z_1$) with elastic system 1 (fig. 1a) from one side is connected to vibroexciter ($O_2x_2y_2z_2$) and from other side – to the material to be displaced ($O_3x_3y_3z_3$) with one-sided connection 2. Free point A_1 of the working member is vectorially connected to the vibratory exciter and its own center of gravity (O_1), just as the point of the material is connected to the O_1 and O_3 points (such connection of the points A_1 and B_1 are used to obtain the mathematical model (9) of spatial motion of the system [11, 17].

Fig. 1c shows an analog of dynamical model (fig.1 a) – vibratory feeder with vibratory exciter and grainy material.

The grainy material (M_3) model (Fig.2) is a cube-shaped body, in which the whole mass is concentrated and equipped with conditional elastic-damping elements ($k_x, k_y, k_z, k_{\square}, k_{\square}, k_{\square}$) unbound from one side (from the side of the working member surface).

They describe the properties of a particular material and vary depending on the working mode of the vibratory machine (moving along the surface of the working body or separately).

Presentation of grainy material in such a way on the one hand allows us to include it in the overall oscillation system (Fig. 1 a), as a solid body (describe spatial motion), and on the other hand describe its deformation (rheological) properties with elastic elements.

The rotational motions of each mass of the system are described in Euler angles (Fig. 1 b), for both towards its own center and from one mass towards another.

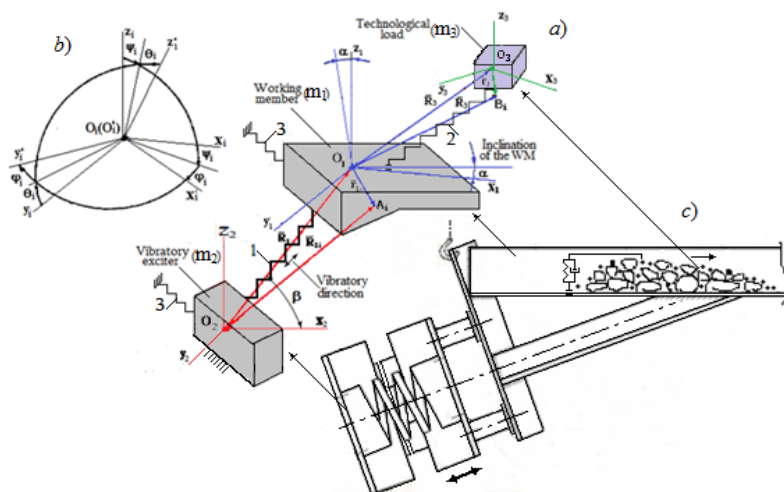


Fig. 1. The dynamical model of vibratory technological machine: a) three mass spatial model, b) rotational motion, c) physical analogue of the model

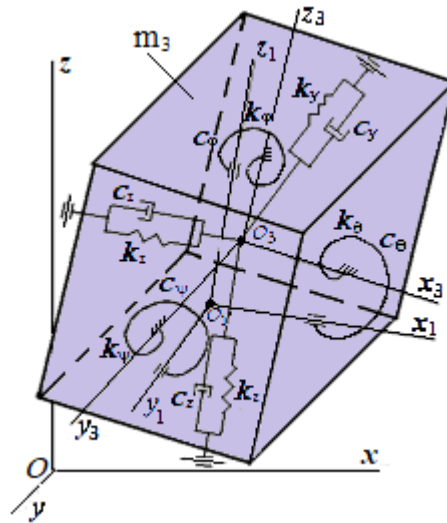


Fig. 2. The spatial model of grainy material

3. Friction force between the working member (m1) and the surfaces of the material (m3)

The problems of the friction force between the working member and the material to be displaced are studied for piece details in detail and substantially [1-3, 20]. They examine the typical forms of frequently used parts (rectangular, spherical, cylindrical, etc.), as well as polyhedral parts with rolling properties for which the friction force is only present at the points of contact of the working member and the load.

Unlike singled out "hard" materials, for grainy materials, there is no definite boundary of point of contact. This view stems from the fact that for dispersive (grainy) materials the detachment from the surface or attachment to it does not happen instantaneously, but along with transitional elastic-damping process. To describe the friction force between the working member and grainy material, different approaches are used [21-25], the essence of which lies in that the reaction of the load on the working member represents the function of load movement and velocity; At the same time, the internal resistance of the material, as well as the resistance of the environment in which the movement takes place (air, liquid, etc.) is taken into account.

$$N_q = f(\dot{q}, q), \quad (1)$$

where q takes the values: $x_{1,3}$, $y_{1,3}$, $z_{1,3}$, and the friction force will be

$$(F_{fr})_q = f_q N_q;$$

When considering the spatial (rectilinear or rotational) motion of a technological load, in addition to the normal reaction, as a result, moments of those forces arise on this or that surface of the working member (Fig. 3):

$$(M_{fr})_q = (F_{fr})_q \cdot r_q, \quad (2)$$

Where f_q is the friction coefficient between the load and the working body (f_q is normally obtained in each cycle of the variable motion depending on the dynamical condition of the load – sliding on the surface, stoppage, etc.); r_q is the distance from friction surface to the center of gravity of the load along the coordinates.

The components of the friction force can be expressed as follows:

$$\begin{aligned} F_{x_3} &= f_x N_z \text{sign}(\dot{x}_3); \\ F_{y_3} &= f_y N_z \text{sign}(\dot{y}_3); \\ F_{z_3} &= f_z N_y \text{sign}(\dot{z}_3), \end{aligned} \quad (3)$$

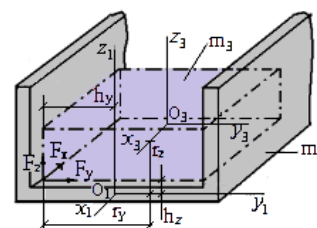


Fig. 3. To determine the friction forces and moments affecting the grainy load

where f_x, f_y, f_z are the friction coefficients between the load and the working member in the directions of x, y, z (subsequently will be obtained: $f_x = f_y = f_z = f$); N_y – normal reaction of the load on the lateral surface (Fig. 3); N_z – normal reaction of the load on the bottom; sign represents a nonlinear function and is determined depending on the sign of velocity V : $\text{sign} = 1$, when $V < 0$ and $\text{sign} = -1$, when $V > 0$.

The moments of the friction forces towards the axes are expressed as follows:

$$\begin{aligned}(M_{fr})_{x_3} &= (F_{z_3}r_y - F_{y_3}r_z)\text{sign}(\dot{\theta}_3); \\ (M_{fr})_{y_3} &= F_{x_3}r_z\text{sign}(\dot{\psi}_3); \\ (M_{fr})_{z_3} &= F_{x_3}r_y\text{sign}(\dot{\varphi}_3);\end{aligned}\quad (4)$$

where r_y, r_z are the distances from friction surface $O_3x_3y_3z_3$ to the system axes. The friction force moments towards the system (working member) $O_1x_1y_1z_1$ are expressed as follows:

$$\begin{aligned}(M_{fr})_{x_1} &= (F_{z_3}h_y - F_{y_3}h_z) \\ (M_{fr})_{y_1} &= F_{x_3}hh_z); \\ (M_{fr})_{z_1} &= F_{x_3}hhh_y);\end{aligned}\quad (5)$$

where h_y, h_z are the distances from the friction surface $O_1x_1y_1z_1$ to the system axes.

In the given example, working member is bound from two sides with the planes: $O_1x_1y_1$, $O_1z_1x_1$, whereas in the direction of O_1x_1 it is open; in this case, on $O_1y_1z_1$ surface the friction force is not present, and therefore, r_x, h_x multiplier members in (4), (5) expressions equal to zero.

4. The excitation force

In a real machine, as a result of initial errors, the excitation force may not be exactly in the center of gravity, but be moved with eccentricities e_x, e_y, e_z (which was mentioned above as well); Besides, when the mass M_1 is dynamically affected, the mass deviates with respect to the external (excitation) force by x_1, y_1, z_1 coordinates at the expense of deformation of the elastic system.

As an illustration, on Fig.4, the condition of M_1 mass is shown before and after the engage-

ment of excitation $Q(t)$ force; M_1 mass is shown in two different conditions – I, II; I corresponds to the initial condition, when $Q(t)$ direction coincides the direction of non-elastic spring axis and passes on the center of gravity OO_1 of M_1 ; II corresponds to the real condition of M_1 mass, i.e. considering the deviations caused by tolerances on the machine manufacturing and installation [3, 5, 23, 26, 27]. The deviations, which are characterized with corners and eccentricities represent the reason of generation of friction forces, which, along with bending deformations of the spring 1, cause the vibration of mass M_1 in space.

The projections of force Q on the axes of coordinate system $O_1x_1y_1z_1$ will be expressed as follows:

$$\begin{aligned}Q_{x_1} &= Q[(\psi_1 - \psi_2)\sin\alpha_1 + \cos\alpha_1]; \\ Q_{y_1} &= Q[\varphi_2 - \varphi_1\cos\alpha_1 - \theta_1\sin\alpha_1]; \\ Q_{z_1} &= Q[(\psi_1 + \psi_2)\cos\alpha_1 + \sin\alpha_1];\end{aligned}\quad (6)$$

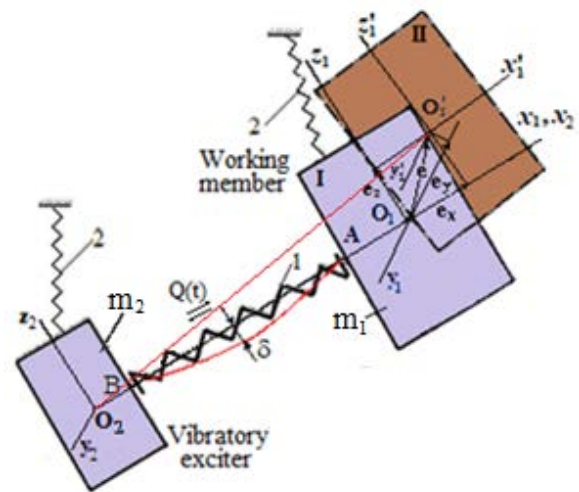


Fig. 4. The position of the working member before and after the engagement of the force

The moments of Q force are defined by the formulas of vector algebra theory [6]; If we define point N , on which Q force vector passes, then the moment of this vector towards $O_1x_1y_1z_1$ coordinate system axes will be as follows:

$$\begin{aligned}M_{x_1} &= e_{y_1}Q_{z_1} - e_{z_1}Q_{y_1}; \\ M_{y_1} &= e_{z_1}Q_{x_1} - e_{x_1}Q_{z_1}; \\ M_{z_1} &= e_{x_1}Q_{y_1} - e_{y_1}Q_{x_1}.\end{aligned}\quad (7)$$

5. Mathematical model of “Grainy material – working member” system spatial vibratory motion

To obtain the differential equation system of load's spatial vibratory movement, the dynamical model presented on Fig.1 was considered. The systemic approach [17] was used, which allows to fully describe the dynamic interaction of masses.

To obtain the full mathematical picture of the material and working member interaction, we used the classical theory [17, 19] of relative (technological load), translational (working member) and absolute (technological load) motion of bodies, which corresponds to the principles of vibratory displacement of one body in relation to the other.

This approach involves obtaining vector equations of velocities of A_i and B_i free points of the masses (relative, translational, absolute) and their expansion on the coordinate axes, then their projection on the coordinate axes of the working member using Euler's angles [17]. Subsequently, by obtaining the functions of the total kinetic and potential energies and resistance, and using the 2nd order Lagrange equation, we obtain the differential equation of spatial motion (9) of each mass.

If we assume that the reaction of the grainy load in the equations of the working member (1) is equal to 0 and also in the left part of the equations we consider only linear constituents, then the system of differential equations of the spatial motion of the working member can vectorially be expressed in this way:

$$M_i \ddot{q}_i + c_i \dot{q}_i + b_i q_i - d_i q_j = A_q Q_q(t), \quad (8)$$

where M_i is the mass of the working member and the moments of inertia during the rectilinear and rotational motion respectively; q_i is the coordinates of spatial motion: $x_1, y_1, z_1, \theta_1, \psi_1, \phi_1$; b_i is the coefficient of resistance towards the axes of spatial coordinates; b_i, d_i coefficients express the interdependence of linear-spatial motions; q_j - the corresponding rotational motion of a linear motion on different axes; F_q - the projection of excitation force and moment on the corresponding axis; A_q - force coefficient on the corresponding axis.

As mentioned, certain structural, physical, and other permissible errors occur during the manufacturing and installation of the vibratory machine [12, 19], which is why the excitation force is not transmitted to the center of gravity of the working body precisely; as a result, the force constituents on the coordinate axes are obtained, as well as the corresponding moments towards the center of gravity. Normally, such deviations are not taken into account due to their smallness, but in resonance machines, they can have a significant influence on the technological process.

After determining the total kinetic and potential energies of the masses M_1 and M_3 and obtaining the analytical expressions (the expansion on the coordinate axes considering Euler's angles [17], working member inclination and angles of vibrations), using the 2nd order Lagrange equation, the system of differential equations of material's spatial motion is obtained:

$$m_3 \ddot{x}_3 + m_3 [(\ddot{x}_1 - \ddot{z}_1 \psi_1) \cos \alpha_1 - (\ddot{z}_1 + \ddot{x}_1 \psi_1) \sin \alpha_1 + \ddot{\psi}_1 z_3 - \ddot{y}_1 \varphi_1 + 2\dot{\psi}_1 \dot{z}_3 - \dot{\varphi}_1 \dot{y} - 2\dot{\varphi}_1 \dot{y}_3] + h_x (\dot{x}_1 \cos \alpha_1 - \dot{z}_1 \sin \alpha_1 + \dot{x}_3) + h_{x_3} \dot{x}_3 - m_3 g (\sin \alpha - \psi_1 \cos \alpha) = - (f_{x_3} N_z + f_{y_3} N_y) \text{sign}(\dot{x}_3),$$

$$m_3 \ddot{y}_3 + m_3 [\ddot{y}_1 + (\ddot{z}_1 \theta_1 - \ddot{x}_1 \varphi_1) \cos \alpha_1 + (\ddot{x}_1 \theta_1 - \ddot{z}_1 \varphi_1) \sin \alpha_1 - \ddot{\theta}_1 z_3 + 2\dot{\theta}_1 \dot{z}_3 + 2\dot{\varphi}_1 \dot{x}_3] + h_y (\dot{y}_1 + \dot{y}_3) + h_{y_3} \dot{y}_3 + k_{y_3} y_3 + m_3 g (\varphi_1 \sin \alpha + \theta_1 \cos \alpha) = -f_{y_3} N_z \text{sign}(\dot{y}_3), \quad (9)$$

$$m_3 \ddot{z}_3 + m_3 [(\ddot{z}_1 + \ddot{x}_1 \psi_1) \cos \alpha_1 + (\ddot{x}_1 - \ddot{z}_1 \psi_1) \sin \alpha_1 - \ddot{y}_1 \theta_1 + \ddot{\theta}_1 y_3 + 2\dot{\theta}_1 \dot{y}_3 - 2\dot{\psi}_1 \dot{x}_3] + h_z (\dot{z}_1 \cos \alpha_1 + \dot{x}_1 \sin \alpha_1 + \dot{z}_3) + h_{x_3} \dot{z}_3 + k_{z_3} z_3 + m_3 g (\cos \alpha - \psi_1 \sin \alpha) = -f_{z_3} N_y \text{sign}(\dot{x}_3),$$

where $\alpha_1 = \alpha + \beta$, α is the inclination of working member towards the horizon and β – the angle of vibration (Fig.1).

In the presented work, with the help of equations (9), this time we consider movement of the material only in the linear spatial directions and study of the influence of various non-working vibrations (y_1 , z_1 , θ_1 , ψ_1 , ϕ_1) of the working member.

In the process of mathematical modelling, the change of the spatial vibrations (strengthening, weakening) occurs not by variation of the vibratory exciter force, but by its own vibration entering resonance in different directions with frequency (vibration) of the constant excitation force and therefore its amplitude changes (increases). Such an approach allows us to investigate and establish the influence of each non-working vibration of the working member on the VTT process, when the working member operates in normal resonance vibratory regime and acts in combination with the aforementioned vibration.

As an example, let's examine the lateral displacement equation (10) of the working member from the vector expression (8). Let's assume that the excitation force changes according to sinusoidal law, with frequency - ω

$$M_1 \ddot{y}_1 + c_y \dot{y}_1 + b_y y_1 - d_\psi \psi_1 = Q_y \sin \omega(t); \quad (10)$$

To amplify it, b_{y_1} should change so that its own frequency ω_{y_1} approaches 50 Hz (should enter in resonance with the excitation force); Such approach allows to observe

the tendency of its influence on the parameters of material displacement (Fig.5).

6. Some of the modeling results

In the figures are given the graphs, where the influence of some partial (of non-working

direction) vibrations on the process of material's displacement is shown.

Fig.5 shows the impact of transverse vibration (y_1) on vertical displacement (z_3) and velocity (V_x); As in other instances (Figs. 6, 7, 8), x_1 is the amplitude of working vibration and as a consequence of the modeling condition, it is constant for each experiment, when ω_{exc} takes the values: 25, 50, 100 Hz (as indicated on the figures).

Fig. 6 shows the change (increase) of vertical partial amplitude (z_1) of the working body and the corresponding changes of dynamical parameters of motion (z_3 , V_x , N_z – reaction of the material on the bottom and N_y – reaction of the material on the lateral surface).

It can also be noticed that different partial vibrations' (y_1 , z_1 , etc.) entry into resonance causes changes in the inertial members associated with it in equations (8) (for example, $\ddot{y}_1 \phi_1$, $\ddot{z}_1 \theta_1$, etc.), which is reflected in the change in material velocity and other dynamical characteristics. (working vibration frequency $\omega_{exc} = 100$ Hz).

Fig. 7 presents the influence of transverse partial vibration (y_1) on the dynamical characteristics of the material, under the conditions of working vibration ($\omega_{exc} = 25$ Hz); z_3 , y_3 are vertical and transverse displacements, V_x – the velocity of displacement in transverse (x_3) direction.

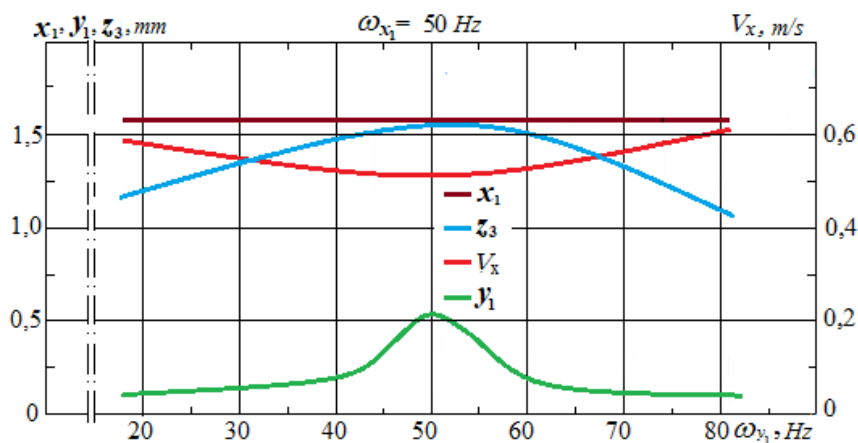


Fig. 5. The dependence of material movement velocity (V_x) and vertical trajectory (z_3) on the transverse vibrations of working member (y_1), when working vibration (x_1) value is constant

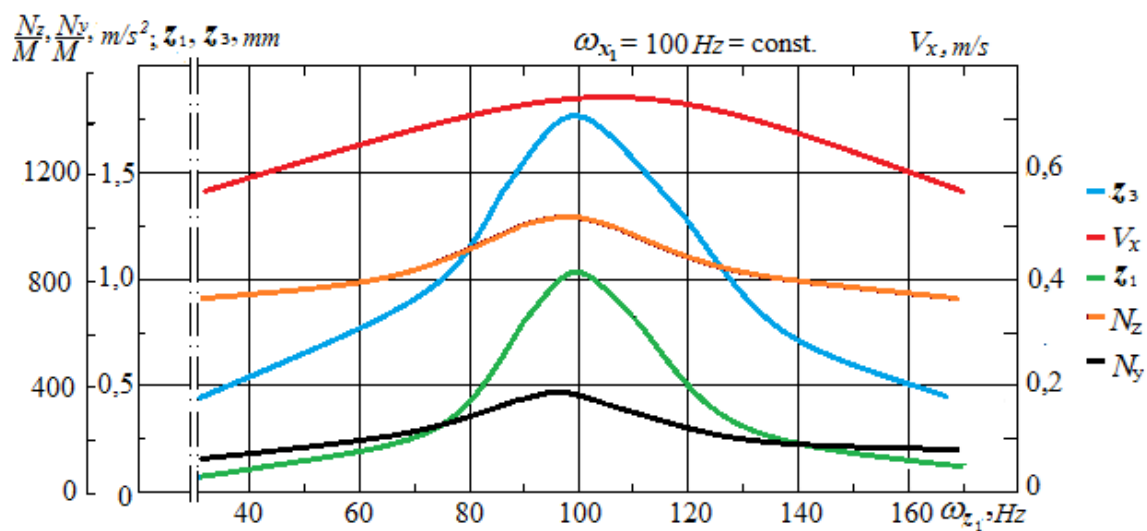


Fig. 6. The dependence of material movement velocity (V_x), vertical trajectory (z_3) and reaction forces (N_z , N_y) on the vertical vibrations of working member (z_1)

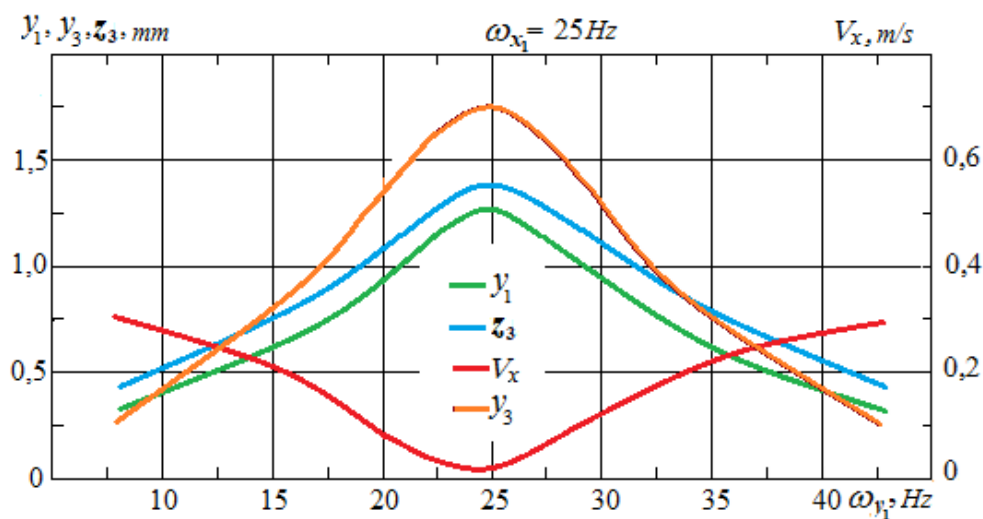


Fig. 7. The dependence of material movement velocity (V_x), vertical trajectory (z_3) and transverse trajectory (y_3) on the transverse vibrations of the working member (y_1)

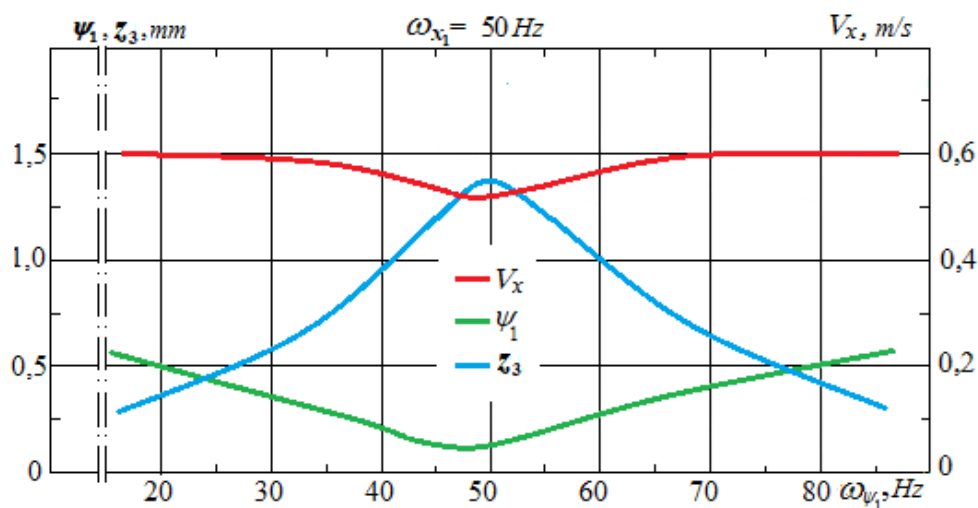


Fig. 8. The dependence of material movement velocity (V_x) and vertical trajectory (z_3) on the rotational vibrations of the working member (ψ_1)

7. Conclusion

1) The modeling has shown that the partial oscillations of resonance vibrofeeder working member, in combination with main (working) vibration, significantly influence the regularity of material displacement; 2) Some partial vibrations (for example, in vertical direction) increase the velocity of material displacement, which indicates the advisability of constructional modernization of the machine; 3) Most partial oscillations have a negative effect on the performance of the machine (displacement velocity reduces), which indicates the need for manufacturers to reduce the tolerances on the accuracy of machine manufacturing and installation.

Acknowledgement

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNS-FG) N FR17_292, “Mathematical modeling of the vibratory technologic processes and design of the new, highly effective machines”

References

- [1] Blekhman I. I., Vibration Mechanics and Vibration Rheology, Theory and applications, Fizmatlit, Moscow, 2018 (in Russian).
- [2] Blekhman I. I., Theory of Vibration Processes and Devices, Vibration mechanics and vibration technology, Ore and metals, SPB, 2013 (in Russian).
- [3] Goncharevich I., Theory of Vibratory Technology, Hemisphere Publisher, 1990.
- [4] Fedorenko I. I., Vibration processes and devices in the agro-industrial complex: monograph. RIO of the Altai State University, Barnaul, 2016 (in Russian).
- [5] Zviadauri V., Dynamics of the vibratory transportation and technological machines; Metsniereba, 2001 (in Russian).
- [6] Panovko G.I., Dynamics of the Vibratory Technological Processes. RCD. M.- Izhevsk, 2006 (in Russian).
- [7] Simsek E., Wirtz S., Scherer V., Kruggel-Emden H., Grochowski R. & Walzel P., An Experimental and Numerical Study of Transversal Dispersion of Granular Material on a Vibrating Conveyor, J. Particulate Science and Technology; V. 26, Issue 2 (2008) 177-196.
- [8] L.Vaisberg, I. Demidov, K. Ivanov., Mechanics of granular materials under vibration action: Methods of description and mathematical modeling Enrichment of ores. № 4, 2015, pp. 21-31, (in Russian).
- [9] Chelomey V.N., (Red.) Vibrations in the technique. Vol. 4, 1981 (in Russian).
- [10] V. Zviadauri. On the approach to the complex research into the vibratory technological process and some factors having an influence on the process regularity, Annals of Agrarian Science, Vol. 17, # 2 (2019) 277-286., <http://journals.org.ge/index.php/aans/issue/archive>.
- [11] Zviadauri V. S., Natriashvili T. M., Tumanishvili G. I., Nadiradze T. N., The features of modeling of the friable material movement along the spatially vibrating surface of the vibratory machine working member. Mechanics of machines, mechanisms and materials, 1 (38), 2017, pp. 21-26.
- [12] Zviadauri V. S., Chelidze M. A., Tumanishvili G. I., On the spatial dynamical model of vibratory displacement, Proceedings of the International Conference of Mechanical Engineering; London, U.K., 2010, pp. 1567 – 1570.
- [13] Xizhong An, Changxing Li., Experiments on densifying packing of equal spheres by two-dimensional vibration, J. Particulate Science and Technology, Vol. 11, Issue 6 (2013) 689-694.
- [14] Czubak P., Vibratory conveyor of the controlled transport velocity with the possibility of the reversal operations, J. of Vibroengineering, Vol. 18, Issue 6 (2016) 3539-3547.
- [15] Sturm Martin, Two-mass linear vibratory conveyor with reduced vibration transmission to the ground. VE International LTD. Vibroengineering Procedia. Vol. 13. 2017, p. 20-23.
- [16] Golovanevskiy V.A., Arsentyev V.A., Blekhman I.I., Vasilkov V.B., Azbel Y.I., Yakimova K., S. Vibration-induced phenomena in bulk granular materials. International Journal of Mineral Processing. Volume 100, Issues 3–4 (2011) 79-85.
- [17] Ganiev R., Kononenko V., Oscillations of solids, Nauka, Moscow, 1976 (in Russian).
- [18] Loktionova O. G., Dynamics of vibratory technological processes and machines for the processing of granular materials, Dissertation, 2008 (in Russian).

- [19] Sloot E.M., Kruyt N.P. Theoretical and experimental study of the transport of granular materials by inclined vibratory conveyors. *Powder Technology* 87(3) (1996) 203-210.
- [20] Ivanov K.S., Optimization of vibrational process, Proc. of the conf. "Vibration Problems" ICOVP, 2011, pp. 174-179.
- [21] M. Chelidze, V. Zviadauri, G. Tumanishvili, A. Gogava, Application of vibration in the wall plastering – covering and cleaning works. *Scientific journal of IFToMM Problems of Mechanics*, # 4(37) (2009) 53-57.
- [22] Gendlina L. I., Levenson S. I., On the problem of the output of cohesive materials by the vibrational way, 2017, pp. 154-158, (in Russian).
- [23] Zeljko V. Despotovic, Milan Lecic, Milan R. Jovic, Ana Djuric, Vibration Control of Resonant, Vibratory feeders with electromagnetic excitation. *FME Transactions*, Vol. 42, No 4, 2014, pp. 281 – 289.
- [24] Khvingia M.V., Dynamics and strength of vibration machines with electromagnetic excitation, *Mashinosroenie*, 1980 (in Russian).
- [25] Ribic A.I., Despotovic Ž.V., High-Performance Feedback Control of Electromagnetic Vibratory Feeder, *IEEE, Transactions on Industrial Electronics*, Vol. 57, Issue: 9, 2010, pp. 3087-3094.
- [26] Mucchi E., Gregorio R., Dalpiaz G., Elastodynamic analysis of vibratory bowl feeders: Modeling and experimental validation. *Mechanism and Machine Theory*, Vol. 60, February, 2013, pp. 60-72.
- [27] Kryukov B. I., Forced oscillations of essentially nonlinear systems, *Mashinostroenie*, Moscow, 1984 (in Russian).